Hyperbolic Functions – Formula Sheet:

Definition of Hyperbolic Functions:

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\sinh(x) = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \ldots\ldots.$$

$$\cosh(x) = x + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots\ldots.$$

Definition of Hyperbolic Functions:

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$\coth(x) = \frac{1}{\tanh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1} \quad x \neq 0$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}$$

Exponential to Hyperbolic:

$$e^x = \cosh(x) + \sinh(x)$$

$$e^{-x} = \cosh(x) - \sinh(x)$$

$$(e^x)^n = [\cosh(x) + \sinh\,(x)]^n$$

$$e^{nx} = [\cosh(nx) + \sinh(nx)]$$

$$e^{-nx} = \cosh(nx) - \sinh(nx)$$

$$e^{2x} = \frac{1 + \tanh(x)}{1 - \tanh(x)}$$

Even – Odd Identities:

$$\sinh(-x) = -\sinh(x)$$

$$\cosh(-x) = \cosh(x)$$

$$\tanh(-x) = -\tanh(x)$$

$$\coth(-x) = -\coth(x)$$

$$\operatorname{csch}(-x) = -\operatorname{csch}(x)$$

$$\operatorname{sech}(-x) = \operatorname{sech}(x)$$

Double Angle Formulas:

$$\sinh(2x) = 2\sinh(x)\cosh(x)$$

$$\cosh(2x) = cosh^2(x) + sinh^2(x)$$
$$\cosh(2x) = 2\,cosh^2(x) - 1$$
$$\cosh(2x) = 2\,sinh^2(x) + 1$$

$$\sinh(2x) = \frac{2\tanh(x)}{1 - tanh^2(x)}$$

$$\cosh(2x) = \frac{1 + tanh^2(x)}{1 - tanh^2(x)}$$

$$\tanh(2x) = \frac{2\tanh(x)}{1 + tanh^2(x)}$$

Pythagorean Identities:

$$cosh^2(x) - sinh^2(x) = 1$$

$$1 - tanh^2(x) = sech^2(x)$$

$$1 - coth^2(x) = -csch^2(x)$$

Power Reducing Formulas:

$$sinh^2(x) = \frac{\cosh(2x) - 1}{2}$$

$$cosh^2(x) = \frac{\cosh(2x) + 1}{2}$$

Triple Angle Formulas:

$$\sinh(3x) = 3\sinh(x) + 4\,sinh^3(x)$$

$$\cosh(3x) = 4\,cosh^3(x) - 3\cosh(x)$$

Power Reducing Formulas:

$$sinh^3(x) = \frac{\sinh(3x) - 3\sinh(x)}{4}$$

$$cosh^3(x) = \frac{\cosh(3x) + 3\cosh(x)}{4}$$

Sum to Product Formulas:

$$\sinh(x) + \sinh(y) = 2\sinh\left(\frac{x+y}{2}\right)\cosh\left(\frac{x-y}{2}\right)$$

$$\sinh(x) - \sinh(y) = 2\cosh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$$

$$\cosh(x) + \cosh(y) = 2\cosh\left(\frac{x+y}{2}\right)\cosh\left(\frac{x-y}{2}\right)$$

$$\cosh(x) - \cosh(y) = 2\sinh\left(\frac{x+y}{2}\right)\sinh\left(\frac{x-y}{2}\right)$$

Product to Sum Formulas:

$$\sinh(x)\sinh(y) = \frac{\cosh(x+y) - \cosh(x-y)}{2}$$

$$\sinh(x)\cosh(y) = \frac{\sinh(x+y) + \sinh(x-y)}{2}$$

$$\cosh(x)\cosh(y) = \frac{\cosh(x+y) + \cosh(x-y)}{2}$$

$$\cosh(x)\sinh(y) = \frac{\sinh(x+y) - \sinh(x-y)}{2}$$

Hyperbolic Inverse Functions:

$$sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right) \qquad D(-\infty, \infty)$$

$$cosh^{-1}(x) = \ln\left(x + \sqrt{x^2-1}\right) \qquad D[1, \infty)$$

$$tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad D(-1, 1)$$

$$coth^{-1}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) \qquad |x| > 1$$

$$sech^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \qquad D(0, 1]$$

$$csch^{-1}(x) = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right) \qquad x \neq 0$$

Derivatives of Hyperbolic Functions:

$$\frac{d}{dx}[\sinh(x)] = \cosh(x)$$

$$\frac{d}{dx}[\cosh(x)] = \sinh(x)$$

$$\frac{d}{dx}[\tanh(x)] = sech^2(x)$$

$$\frac{d}{dx}[\coth(x)] = -csch^2(x)$$

$$\frac{d}{dx}[\operatorname{csch}(x)] = -\operatorname{csch}(x)\coth(x)$$

$$\frac{d}{dx}[\operatorname{sech}(x)] = -\operatorname{sech}(x)\tanh(x)$$

Derivatives of Inverse Hyperbolic Functions:	**Integral of Hyperbolic Functions:**
$\frac{d}{dx}[sinh^{-1}(u)] = \frac{u'}{\sqrt{u^2+1}}$	$\int \cosh(u)\, du = \sinh(u) + C$
$\frac{d}{dx}[cosh^{-1}(u)] = \frac{u'}{\sqrt{u^2-1}}$	$\int \sinh(u)\, du = \cosh(u) + C$
$\frac{d}{dx}[tanh^{-1}(u)] = \frac{u'}{1-u^2}$	$\int sech^2(u)\, du = \tanh(u) + C$
$\frac{d}{dx}[coth^{-1}(u)] = \frac{u'}{1-u^2}$	$\int csch^2(u)\, du = -\coth(u) + C$
$\frac{d}{dx}[sech^{-1}(u)] = \frac{-u'}{u\sqrt{1-u^2}}$	$\int sech\,(u)\tanh(u)\, du = -\operatorname{sech}(u) + C$
$\frac{d}{dx}[csch^{-1}(u)] = \frac{-u'}{\lvert u\rvert\sqrt{1+u^2}}$	$\int \operatorname{csch}(u)\coth(u)\, du = -\operatorname{csch}(u) + C$
	$\int \tanh(u)\, du = \ln\lvert\cosh(u)\rvert + C$
	$\int \coth(u)\, du = \ln\lvert\sinh(u)\rvert + C$
	$\int \operatorname{sech}(u)\, du = tan^{-1}\lvert\sinh(u)\rvert + C$
	$\int \operatorname{csch}(u)\, du = \ln\left\lvert\tanh\left(\frac{u}{2}\right)\right\rvert + C$