**Projectile Motion Formula Sheet:**

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|  | **Vector Formulas:**$$v\_{x}=v\cos(θ) v\_{y}=v\sin(θ)$$$$v=\sqrt{v\_{x}^{2}+v\_{y}^{2}} θ=tan^{-1}\left(\frac{v\_{y}}{v\_{x}}\right) $$ |
|  **Facts to Know**1. $v\_{o}=v\_{x}=constant$
2. $v\_{yo}=0$
3. $a\_{y}=-g=-9.8 m/s^{2}$
4. $g=+9.8 m/s^{2}$
5. $θ=0° and\cos(0°=1)$

**Derived Equations:**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**Initial Velocity:**$$v\_{o}=R\sqrt{\frac{g}{2h}}$$**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_****Range Vs Time:**$$\frac{R\_{1}}{t\_{1}}=\frac{R\_{2}}{t\_{2}}$$ | **Height of the Cliff/Building:** $$h=\frac{1}{2}gt^{2} h=\frac{R^{2}g}{2v\_{o}^{2}}$$ |
| **Range:**  $\cos(0°=1) and v\_{o}=v\_{x} $$$R= v\_{x}t R=v\_{o}\cos(θ)t R=v\_{o}\sqrt{\frac{2h}{g}} $$ |
| **Displacement:**$$d\_{x}=x\_{F}-x\_{o} d\_{y}=y\_{F}-y\_{o}$$$$d\_{x}=v\_{o}\cos(θ)t d\_{y}=v\_{o}\sin(θ)t-\frac{1}{2}gt^{2}$$$$d= v\_{o}t\sqrt{1+\left(\frac{gt}{2v\_{o}}\right)^{2}} d=\sqrt{d\_{x}^{2}+d\_{y}^{2}}$$ |
| **Final Vertical Velocity:** $$v\_{yF}=-gt$$ |
| **Final Velocity:**$$v\_{F}=\sqrt{v\_{o}^{2}+\left(gt\right)^{2}} v\_{F}=\sqrt{v\_{o}^{2}-2gd\_{y}} θ\_{F}=cos^{-1}\left(\frac{v\_{o}}{v\_{f}}\right)$$**Note**: *dy is negative when the projectile is falling. (g = +9.8)* |
| **Time to Reach the Ground:**$$t=\sqrt{\frac{2h}{g}}=\frac{R}{v\_{o}}=\frac{v\_{yF}}{-g}=\frac{\sqrt{v\_{F}^{2}-v\_{o}^{2}}}{g}$$ |

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|  | **Point A:** $$v\_{x}=v\_{o}\cos(θ) v\_{yo}=v\_{o}\sin(θ)$$$$v\_{y} decreases by 9.8 {m}/{s} every second.$$$$v\_{x} is constant.$$**Point B:** $$v\_{y}=0 v=v\_{x}$$**All Points:**$$a\_{y}=-g= -9.8 m/s^{2}$$$$g=+9.8 m/s^{2}$$ |
| **Height:**$$H= \frac{v\_{o}^{2}sin^{2}\left(θ\right)}{2g}=\frac{R\tan(θ)}{4}=\frac{v\_{yo}^{2}}{2g}$$$$H\_{Max}=\frac{v\_{o}^{2}}{2g} when θ=90°$$ | **Range:**$$R=\frac{v\_{o}^{2}\sin(\left(2θ\right))}{g}=\frac{2v\_{x}v\_{yo}}{g}=\frac{4H}{\tan(θ)}$$$$R\_{Max}=\frac{v\_{o}^{2}}{g} when θ=45°$$ |
| **Time of Flight:**$$t\_{A\rightarrow B}=\frac{v\_{o}\sin(θ)}{g} t\_{A\rightarrow C}= \frac{2v\_{o}\sin(θ)}{g} $$ | **Initial Angle:**$$θ\_{o}=tan^{-1}\left(\frac{4H}{R}\right)=sin^{-1}\left(\sqrt{\frac{2gH}{v\_{o}^{2}}}\right)=\frac{1}{2}sin^{-1}\left(\frac{Rg}{v\_{o}^{2}}\right)$$ |
| **Velocity Components:**$$v\_{x}=v\_{o}\cos(θ) v\_{y}=v\_{o}\sin(θ)-gt$$ | **Equation of Trajectory:**$$y=x\tan(θ)-\frac{gx^{2}}{2v\_{o}^{2}cos^{2}(θ)}$$**Note:** $x=dx and y=dy if \left(x\_{o},y\_{o}\right) is \left(0, 0\right).$ |
| **Initial Velocity:** $$v\_{o}=\sqrt{\frac{Rg}{\sin((2θ))}}=\frac{\sqrt{2gH}}{\sin(θ)}=\frac{gt\_{A\rightarrow C}}{2\sin(θ)}$$ | **Final Velocity:**$$v\_{F}=\sqrt{v\_{xF}^{2}+v\_{yF}^{2}}$$$$v\_{F}=\sqrt{v\_{o}^{2}-2gtv\_{o}\sin(θ)+(gt)^{2}}$$ |
| **Position:**$$x\_{F}=x\_{o}+v\_{o}\cos(θ)t$$$$y\_{F}=y\_{0}+v\_{o}\sin(θ)t-\frac{1}{2}gt^{2}$$**Position Vector**: $\left(x\_{o},y\_{o}\right)\rightarrow \left(0, 0\right) $$$\vec{r}=\left[v\_{o}\cos(θ)t\right]i+\left[v\_{o}\sin(θ)t-{1}/{2}gt^{2}\right]j$$$$\vec{r}=r\_{x}i+r\_{y}j \left|\vec{r}\right|=\sqrt{r\_{x}^{2}+r\_{y}^{2}}$$ | **Displacement:**$$d\_{x}=v\_{o}\cos(θ)t d\_{y}=v\_{o}\sin(θ)t-\frac{1}{2}gt^{2}$$$$d=\sqrt{d\_{x}^{2}+d\_{y}^{2}}$$$$d= v\_{o}t\sqrt{1-\frac{gt\sin(θ)}{v\_{o}}+\left(\frac{gt}{2v\_{o}}\right)^{2}}=|\vec{r}| $$ |

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|  | **Point A:** $$v\_{x}=v\_{o}\cos(θ) v\_{yo}=v\_{o}\sin(θ)$$$$v\_{y} decreases by 9.8 {m}/{s} every second.$$$$v\_{x} is constant.$$**Point B:** $$v\_{y}=0 v=v\_{x}$$**All Points:**$$a\_{y}=-g= -9.8 m/s^{2}$$$$g=+9.8 m/s^{2} and h=y\_{o} $$ |
| **Height:**$$H\_{max}=h+H= y\_{o}+\frac{v\_{o}^{2}sin^{2}(θ)}{2g}$$ | **Range:**$$R= v\_{o}\cos(θ t)$$ |
| **Time of Flight:**$$t\_{A\rightarrow B}=\frac{v\_{o}\sin(θ)}{g} t\_{B\rightarrow C}=\sqrt{\frac{2H\_{max}}{g}}$$$$t\_{A\rightarrow C}= \frac{v\_{o}\sin(θ)}{g}+\sqrt{\frac{2H\_{max}}{g}} $$$$t\_{A\rightarrow C}=\frac{v\_{o}\sin(θ+\sqrt{v\_{o}^{2}sin^{2}\left(θ\right)+2gy\_{o}})}{g}$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**Equation of Trajectory:**$$y=x\tan(θ)-\frac{gx^{2}}{2v\_{o}^{2}cos^{2}\left(θ\right)}$$**Note:** $x=dx and y=dy if \left(x\_{o},y\_{o}\right) is \left(0, 0\right).$ | **Displacement:**$$d\_{x}=v\_{o}\cos(θ)t d\_{y}=v\_{o}\sin(θ)t-\frac{1}{2}gt^{2}$$$$d=\sqrt{d\_{x}^{2}+d\_{y}^{2}}$$$$d= v\_{o}t\sqrt{1-\frac{gt\sin(θ)}{v\_{o}}+\left(\frac{gt}{2v\_{o}}\right)^{2}}=|\vec{r}| $$$$d\_{x}=x\_{F}-x\_{o} d\_{y}=y\_{F}-y\_{o}$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**Position:**$$x\_{F}=x\_{o}+v\_{o}\cos(θ)t$$$$y\_{F}=y\_{0}+v\_{o}\sin(θ)t-\frac{1}{2}gt^{2}$$ |
| **Velocity Components:**$$v\_{x}=v\_{o}\cos(θ) v\_{y}=v\_{o}\sin(θ)-gt$$ | **Position Vector**: $$\vec{r}=\left[v\_{o}\cos(θ)t\right]i+\left[v\_{o}\sin(θ)t-{1}/{2}gt^{2}\right]j$$ |
| **Initial Velocity:**$$v\_{o}=\frac{R}{t\cos(θ)}=\frac{\sqrt{2g(H\_{max}-h)}}{\sin(θ)}=\frac{{1}/{2}gt^{2}-y\_{o}}{t\sin(θ)}$$**Final Velocity:**$$v\_{F}=\sqrt{v\_{xF}^{2}+v\_{yF}^{2}} θ\_{F}=cos^{-1}\left(\frac{v\_{o}\cos(θ\_{o})}{v\_{f}}\right) $$$$v\_{F}=\sqrt{v\_{o}^{2}-2gtv\_{o}\sin(θ)+(gt)^{2}} θ\_{F}=tan^{-1}\left(\frac{v\_{y}}{v\_{x}}\right) $$ | **Initial Angle:**$$θ\_{o}=cos^{-1}\left(\frac{R}{v\_{o}t}\right)=sin^{-1}\left(\frac{\sqrt{2g(H\_{max}-y\_{o)}}}{v\_{o}}\right)$$$$θ\_{o}=sin^{-1}\left(\frac{{1}/{2}gt^{2}-y\_{o}}{v\_{o}t}\right)=tan^{-1}\left(\frac{{1}/{2}gt^{2}-y\_{o}}{R}\right)$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**Height of the Building:**$$h=\left|d\_{y}\right|=\left|v\_{o}\sin(θ)t-\frac{1}{2}gt^{2}\right|$$ |