Matrices – Formula Sheet:

Systems of Equations – 2 Variables:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

$$\boldsymbol{x} = \frac{c_1b_2 - b_1c_2}{a_1b_2 - b_1a_2}$$

$$\boldsymbol{y} = \frac{a_1c_2 - c_1a_2}{a_1b_2 - b_1a_2}$$

Cramer's Rule – 2x2 Matrices with Determinants:

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - b_1a_2$$

$$D_x = \begin{vmatrix} \boldsymbol{c_1} & b_1 \\ \boldsymbol{c_2} & b_2 \end{vmatrix} = c_1b_2 - b_1c_2$$

$$D_y = \begin{vmatrix} a_1 & \boldsymbol{c_1} \\ a_2 & \boldsymbol{c_2} \end{vmatrix} = a_1c_2 - c_1a_2$$

$$\boldsymbol{x} = \frac{D_x}{D} \qquad \boldsymbol{y} = \frac{D_y}{D}$$

Systems of Equations – 3 Variables:

$$a_1x + b_1y + c_1z = \boldsymbol{d_1}$$
$$a_2x + b_2y + c_2z = \boldsymbol{d_2}$$
$$a_3x + b_3y + c_3z = \boldsymbol{d_3}$$

$$\boldsymbol{x} = \frac{d_1(b_2c_3 - c_2b_3) - b_1(d_2c_3 - c_2d_3) + c_1(d_2b_3 - b_2d_3)}{a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)}$$

$$\boldsymbol{y} = \frac{a_1(d_2c_3 - c_2d_3) - d_1(a_2c_3 - c_2a_3) + c_1(a_2d_3 - d_2a_3)}{a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)}$$

$$\boldsymbol{z} = \frac{a_1(b_2d_3 - d_2b_3) - b_1(a_2d_3 - d_2a_3) + d_1(a_2b_3 - b_2a_3)}{a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)}$$

Cramer's Rule – 3x3 Matrices with Determinants:

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$D = a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$$

$$D_x = \begin{vmatrix} \boldsymbol{d_1} & b_1 & c_1 \\ \boldsymbol{d_2} & b_2 & c_2 \\ \boldsymbol{d_3} & b_3 & c_3 \end{vmatrix} = d_1(b_2c_3 - c_2b_3) - b_1(d_2c_3 - c_2d_3) + c_1(d_2b_3 - b_2d_3)$$

$$D_y = \begin{vmatrix} a_1 & \boldsymbol{d_1} & c_1 \\ a_2 & \boldsymbol{d_2} & c_2 \\ a_3 & \boldsymbol{d_3} & c_3 \end{vmatrix} = a_1(d_2c_3 - c_2d_3) - d_1(a_2c_3 - c_2a_3) + c_1(a_2d_3 - d_2a_3)$$

$$D_z = \begin{vmatrix} a_1 & b_1 & \boldsymbol{d_1} \\ a_2 & b_2 & \boldsymbol{d_2} \\ a_3 & b_3 & \boldsymbol{d_3} \end{vmatrix} = a_1(b_2d_3 - d_2b_3) - b_1(a_2d_3 - d_2a_3) + d_1(a_2b_3 - b_2a_3)$$

$$\boldsymbol{x} = \frac{D_x}{D} \qquad \boldsymbol{y} = \frac{D_y}{D} \qquad \boldsymbol{z} = \frac{D_z}{D} \qquad D \neq 0$$

Matrix Addition: $A + B = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{vmatrix} a+e & b+f \\ c+g & d+h \end{vmatrix}$	**Matrix Subtraction:** $A - B = \begin{vmatrix} a & b \\ c & d \end{vmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{vmatrix} a-e & b-f \\ c-g & d-h \end{vmatrix}$
Scalar Multiplication: $nA = n \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} na & nb \\ nc & nd \end{vmatrix}$	**Matrix Multiplication:** $AB = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{vmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{vmatrix}$ $2\ Rows\ x\ 2\ Columns = 2x2\ Matrix$
MM: $2\ Rows\ x\ 2\ Columns = 2x2\ Matrix$ $AB = \begin{vmatrix} a & b & c \\ d & e & f \end{vmatrix} \begin{vmatrix} g & h \\ i & j \\ k & l \end{vmatrix}$ $AB = \begin{vmatrix} ag+bi+ck & ah+bj+cl \\ dg+ei+fk & dh+ej+fl \end{vmatrix}$	**MM:** $1\ Row\ x\ 1\ Column = 1x1\ Matrix$ $AB = \begin{vmatrix} a & b & c \end{vmatrix} \begin{vmatrix} d \\ e \\ f \end{vmatrix}$ $AB = \begin{vmatrix} ad+be+cf \end{vmatrix}$
MM: $3\ Rows\ x\ 3\ Columns = 3x3\ Matrix$ $AB = \begin{vmatrix} a \\ b \\ c \end{vmatrix} \begin{vmatrix} d & e & f \end{vmatrix}$ $AB = \begin{vmatrix} ad & ae & af \\ bd & be & bf \\ cd & ce & cf \end{vmatrix}$	**MM:** $2\ Rows\ x\ 4\ Columns = 2x4\ Matrix$ $AB = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} e & f & g & h \\ i & j & k & l \end{vmatrix}$ $AB = \begin{vmatrix} ae+bi & af+bj & ag+bk & ah+bl \\ ce+di & cf+dj & cg+dk & ch+dl \end{vmatrix}$
MM: $3\ Rows\ x\ 3\ Columns = 3x3\ Matrix$ $AB = \begin{vmatrix} a & b \\ c & d \\ e & f \end{vmatrix} \begin{vmatrix} g & h & i \\ j & k & l \end{vmatrix}$ $AB = \begin{vmatrix} ag+bj & ah+bk & ai+bl \\ cg+dj & ch+dk & ci+dl \\ eg+fj & eh+fk & ei+fl \end{vmatrix}$	**MM:** $3\ Rows\ x\ 3\ Columns = 3x3\ Matrix$ $AB = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{vmatrix} j & k & l \\ m & n & o \\ p & q & r \end{vmatrix}$ $AB = \begin{vmatrix} aj+bm+cp & ak+bn+cq & al+bo+cr \\ dj+em+fp & dk+en+fq & dl+eo+fr \\ gj+hm+ip & gk+hn+iq & gl+ho+ir \end{vmatrix}$

Multiplicative Identity Matrices:	Multiplicative Inverse of a 2x2 Matrix:
$I_2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ $I_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ $AA^{-1} = I_n \qquad A^{-1}A = I_n$	$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $A^{-1} = \frac{1}{ad - bc} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$ **Note:** $If\ ad - bc = 0, then\ A\ does\ not\ have\ a\ multiplicative\ inverse.$

Multiplicative Inverse of a 3x3 Matrix:

$$A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|} \begin{vmatrix} \begin{vmatrix} e & f \\ h & i \end{vmatrix} & \begin{vmatrix} c & b \\ i & h \end{vmatrix} & \begin{vmatrix} b & c \\ e & f \end{vmatrix} \\ \begin{vmatrix} f & d \\ i & g \end{vmatrix} & \begin{vmatrix} a & c \\ g & i \end{vmatrix} & \begin{vmatrix} c & a \\ f & d \end{vmatrix} \\ \begin{vmatrix} d & e \\ g & h \end{vmatrix} & \begin{vmatrix} b & a \\ h & g \end{vmatrix} & \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|} \begin{vmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{vmatrix}$$